Chapter 14: Wave Motion
Thursday April 2nd

- Introduction to wave motion
    - Definitions
- The wave equation
- Energy in SHM
- Energy in waves
- Wave superposition
    - Interference
- Examples, demonstrations and *i*clicker

- Final Mini Exam next week on Thursday (April 9)
- Will cover oscillations and waves (this week/next LONCAPA)

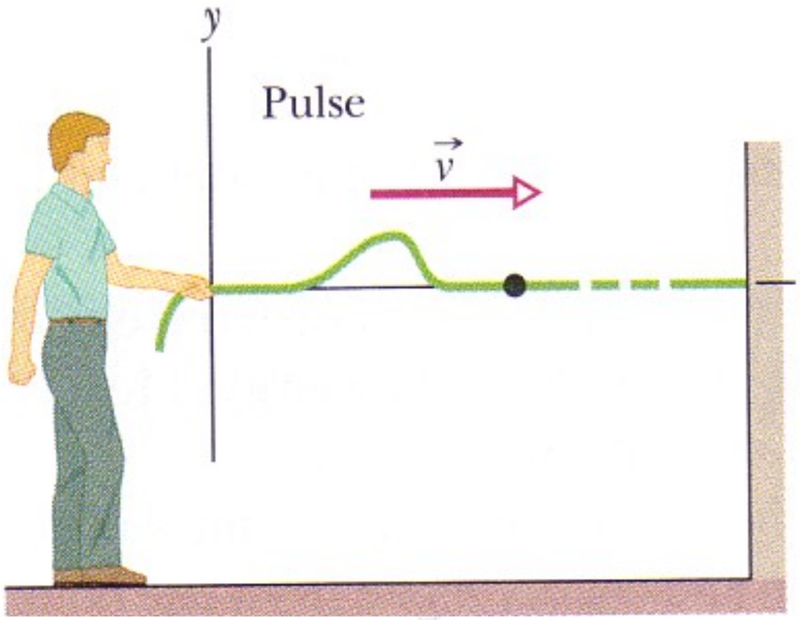
Reading: up to page 234 in Ch. 14

Waves I - types of waves

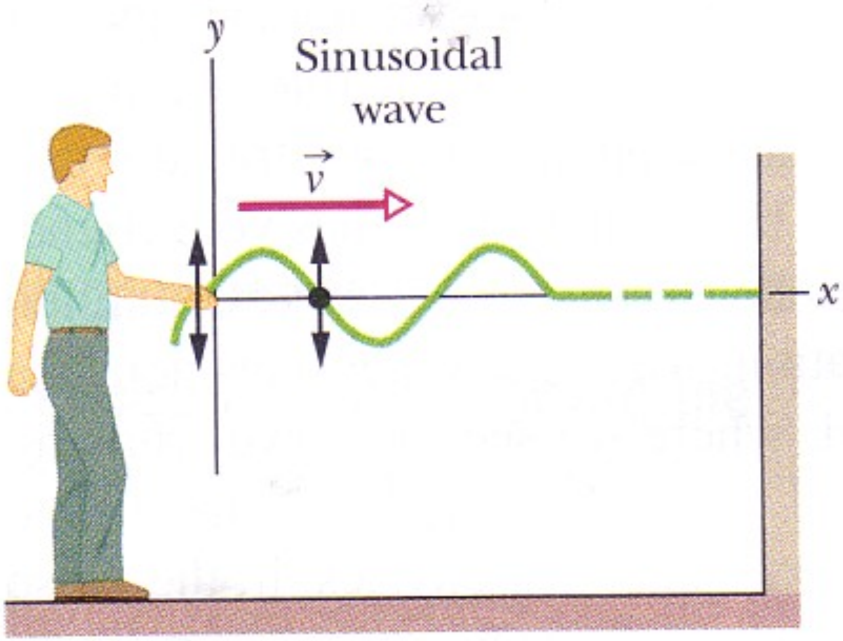
1. Mechanical waves: water waves, sound waves, seismic waves.
2. Electromagnetic waves: radio waves, visible light, ultraviolet light, x-rays, gamma rays.
3. Matter waves: electrons, protons, neutrons, anti-protons, *etc.*.

1. These are the most familiar. We encounter them every day. The common feature of all mechanical waves is that they are governed entirely by Newton's laws, and can exist only within a material medium.
2. All electromagnetic waves travel through vacuum at the same speed c, the speed of light, where c = 299 792 458 m/s. Electromagnetic waves are governed by Maxwell's equations (PHY 2049).
3. Although one thinks of matter as being made up from particles, one can also describe these particles as waves. Matter waves are governed by the laws of quantum mechanics, or the Schrödinger and Dirac equations.

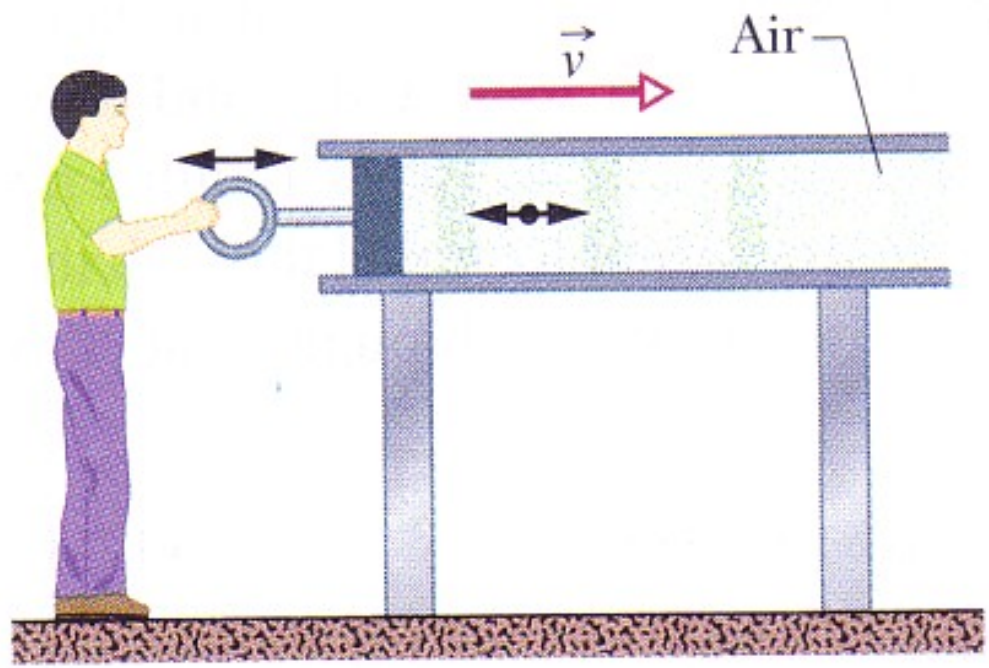
Waves I - types of waves



Transverse waves
(2 polarizations)

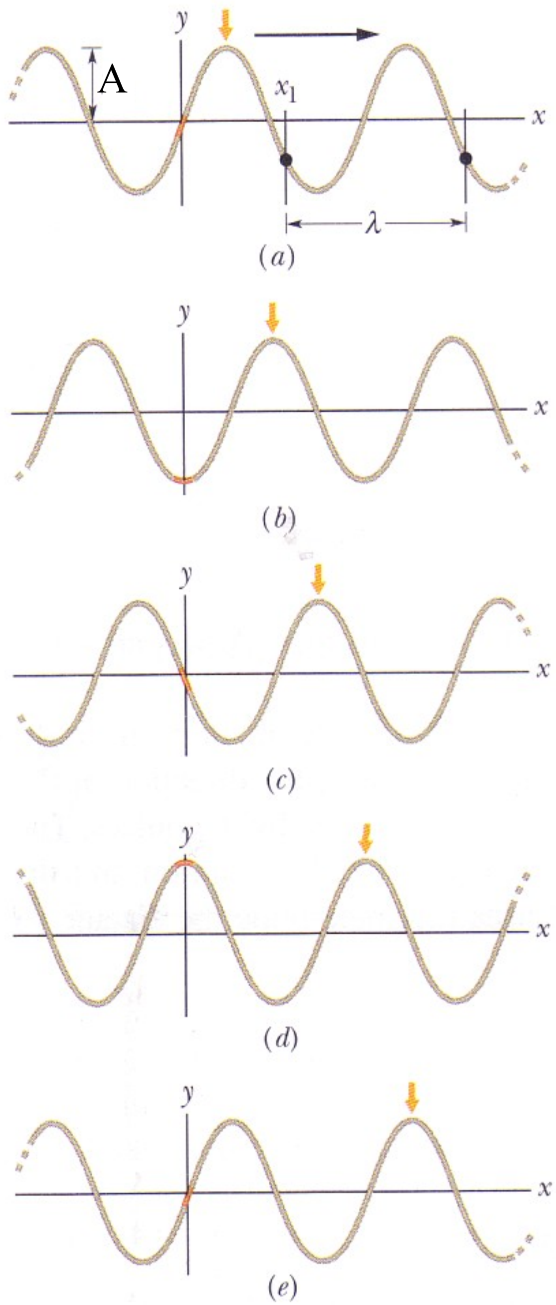


Longitudinal waves



Waves I - wavelength and frequency

Transverse sinusoidal wave



Amplitude

Displacement

$$y(x,t) = \mathrm{A}\cos(\overbrace{kx \pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

Wavelength (consider wave at $t = 0$):

$$y(x,0) = \mathrm{A}\sin kx = \mathrm{A}\sin k\left(x + \lambda\right)$$

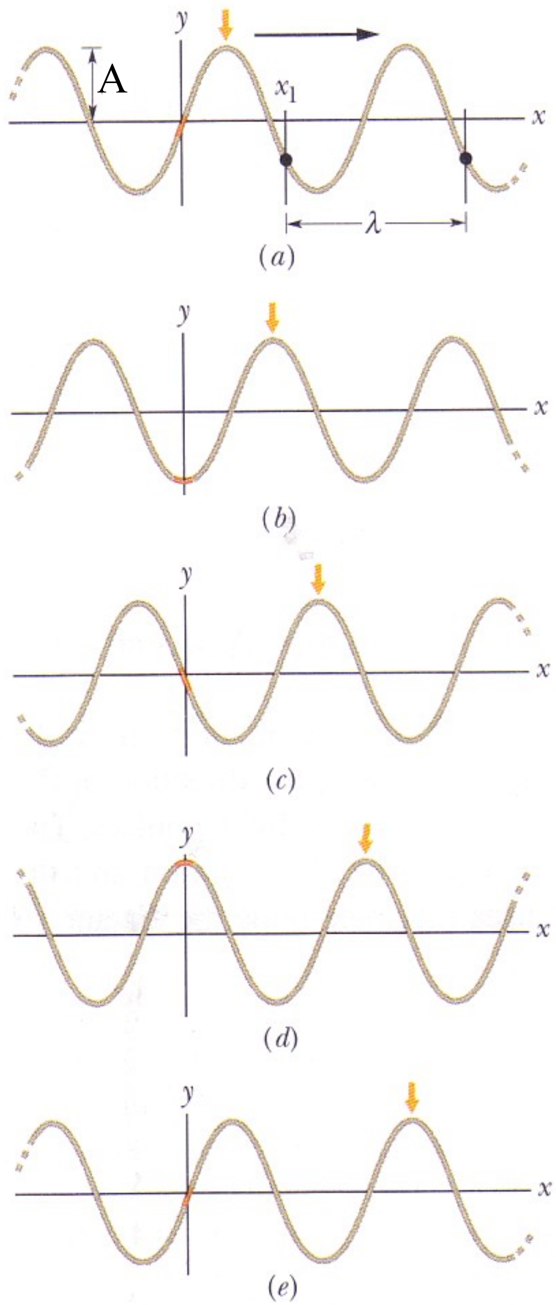
$$= \mathrm{A}\sin\left(kx + k\lambda\right)$$

You can always add 2π to the phase of a wave without changing its displacement, *i.e.*,

$$k\lambda = 2\pi \qquad \text{or} \qquad k = \frac{2\pi}{\lambda}$$

Waves I - wavelength and frequency

Transverse sinusoidal wave



Displacement

Amplitude

$$y(x,t) = A\cos(\overbrace{kx \pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

Wavelength (consider wave at $t = 0$):

$$k\lambda = 2\pi \qquad \text{or} \qquad k = \frac{2\pi}{\lambda}$$

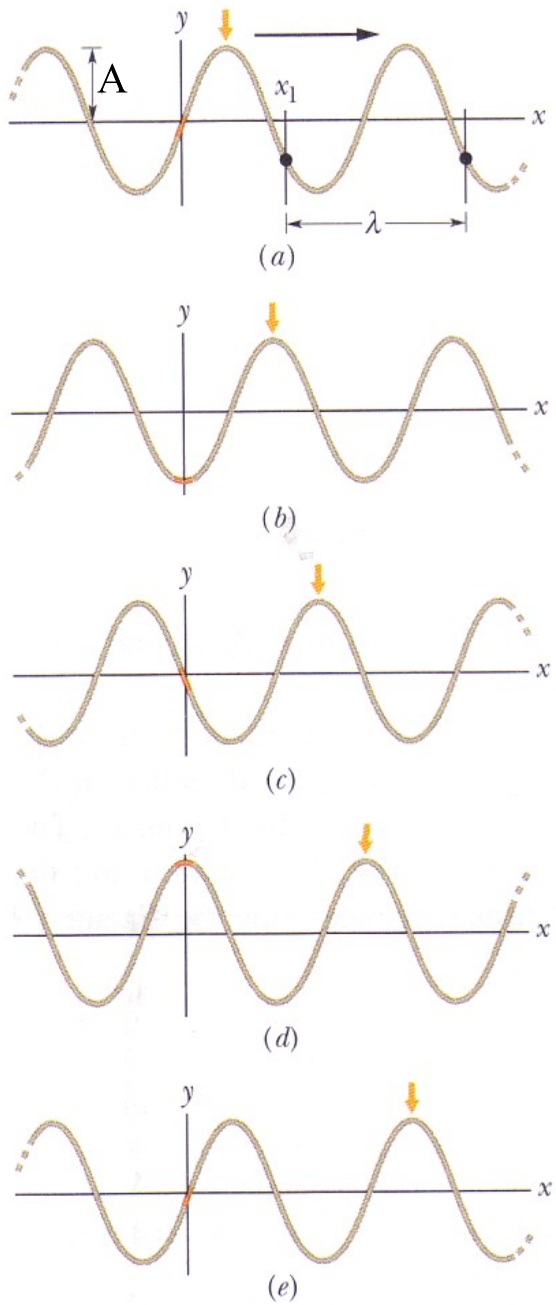
We call k the angular wavenumber.

The SI unit is radian per meter, or meter^{-1}.

This k is NOT the same as spring constant!!!

Waves I - wavelength and frequency

Transverse sinusoidal wave



Amplitude

Displacement

Phase

$$y(x,t) = A\cos(kx \overbrace{\pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

Period and frequency (consider wave at $x = 0$):

$$y(0,t) = A\sin\left(-\omega t\right) = -A\sin\omega t$$

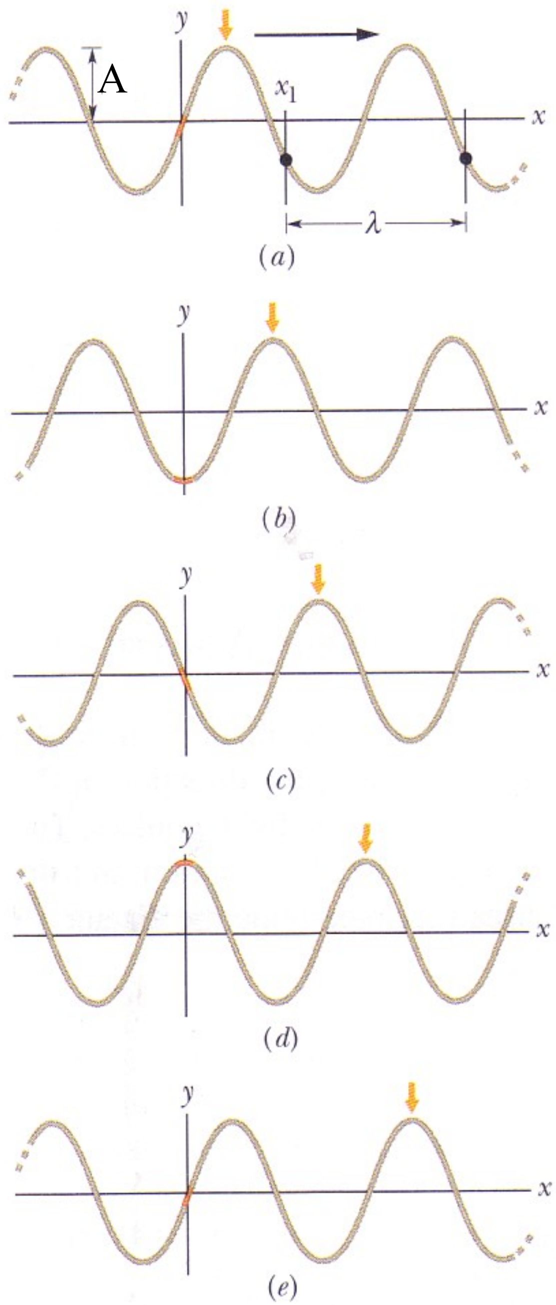
$$= -A\sin\omega\left(t + T\right)$$

Again, we can add 2π to the phase,

$$\omega T = 2\pi \qquad \text{or} \qquad T = \frac{2\pi}{\omega}$$

Waves I - wavelength and frequency

Transverse sinusoidal wave



Displacement

Amplitude

$$y(x,t) = A\cos(\overbrace{kx \pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

Period and frequency (consider wave at $x = 0$):

$$\omega T = 2\pi \qquad \text{or} \qquad T = \frac{2\pi}{\omega}$$

We call π the angular frequency.

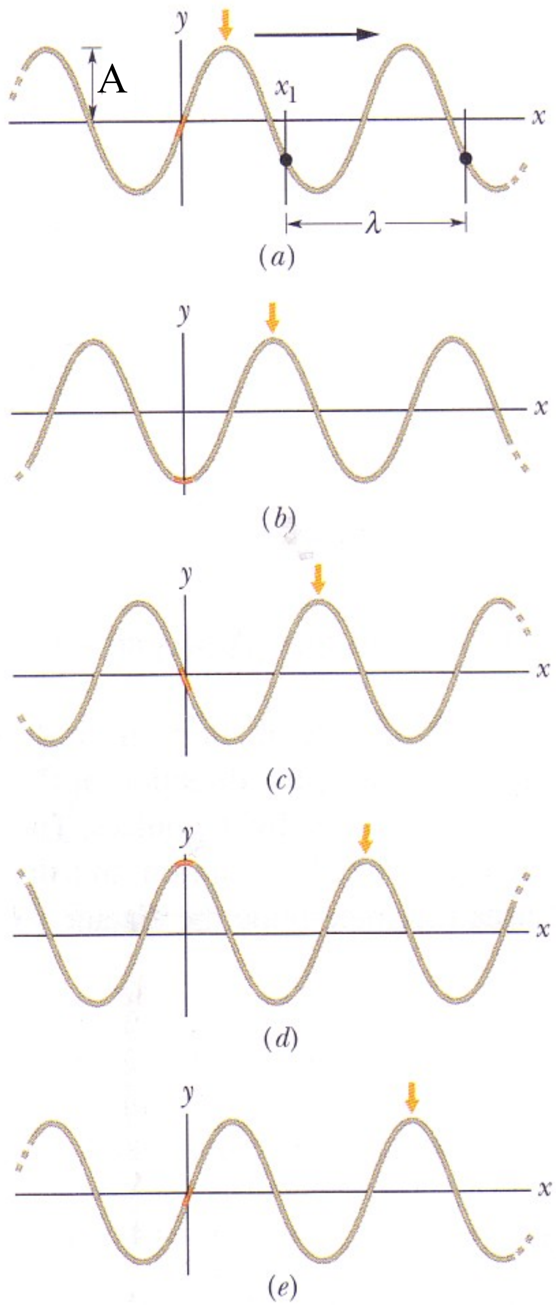
The SI unit is radian per second.

The frequency f is defined as $1/T$.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The speed of a traveling wave

Transverse sinusoidal wave



- A fixed point on a wave has a constant value of the phase (orange arrow), *i.e.*,

$$kx - \omega t = \text{constant}$$

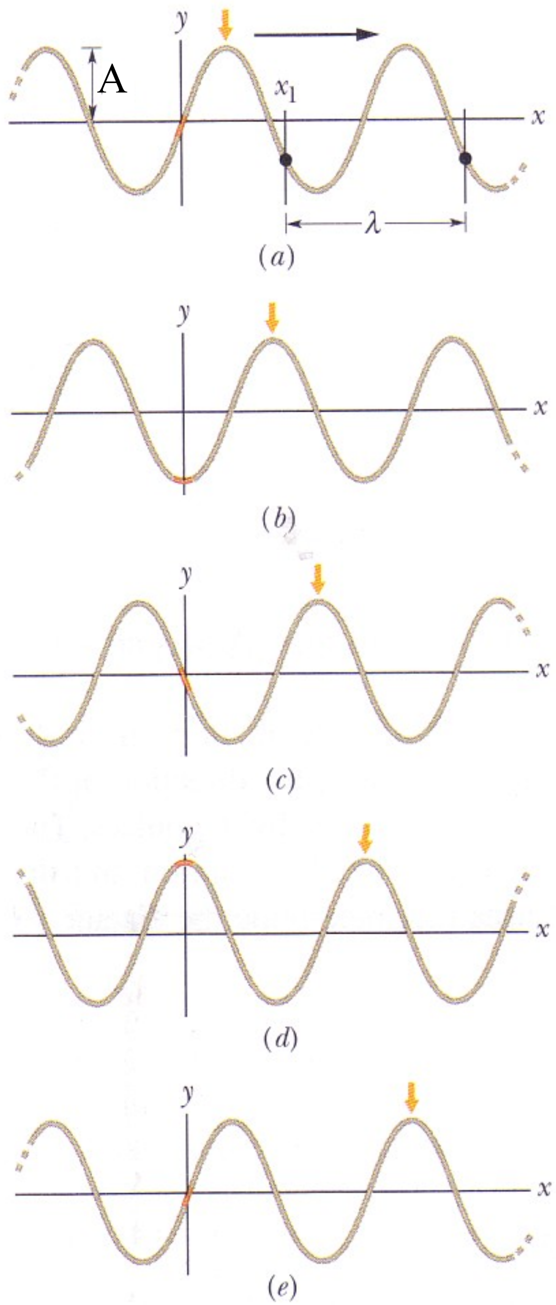
$$\Rightarrow k\frac{dx}{dt} - \omega = 0 \qquad \text{or} \qquad \frac{dx}{dt} = v = \frac{\omega}{k}$$

Or

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = f\lambda$$

The speed of a traveling wave

Transverse sinusoidal wave



- For a wave traveling in the opposite direction, we simply set time to run backwards, *i.e.*, replace t with $-t$.

$$kx + \omega t = \text{constant}$$

$$\Rightarrow \; k\frac{dx}{dt} + \omega = 0 \qquad \text{or} \qquad \frac{dx}{dt} = v = -\frac{\omega}{k}$$

$$y(x,t) = \mathrm{A}\sin\left(kx + \omega t\right)$$

- So, general sinusoidal solution is:

$$y(x,t) = \mathrm{A}\sin\left(kx \pm \omega t\right)$$

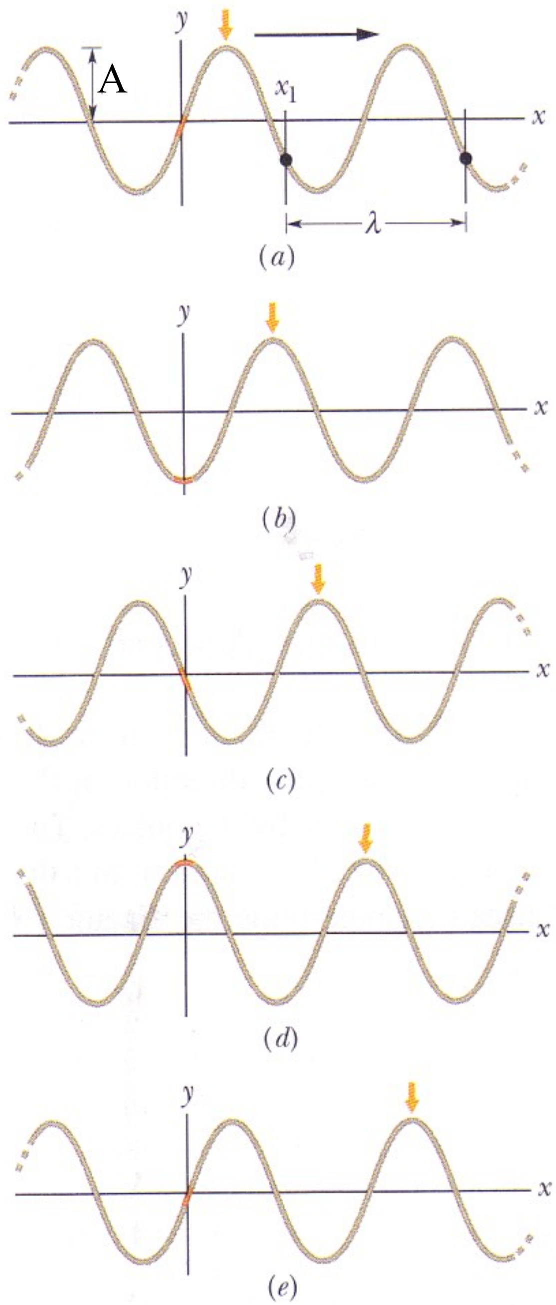
- In fact, any function of the form

$$y(x,t) = \mathrm{A}\times f^{n}\left(kx \pm \omega t\right)$$

is a solution.

Review - wavelength and frequency

Transverse wave



Displacement

Amplitude

Phase

$$y(x,t) = A\cos(\overbrace{kx \pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

$$k = \frac{2\pi}{\lambda}$$

k is the angular wavenumber, λ is the wavelength.

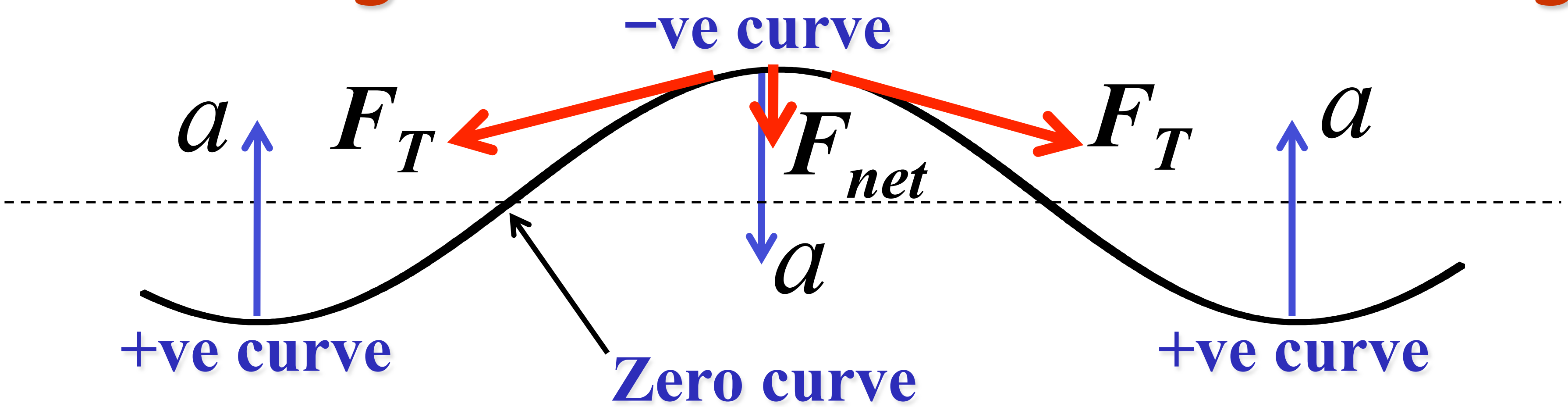
$$\omega = \frac{2\pi}{T}$$

ω is the angular frequency.

frequency $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

velocity $$v = \mp\frac{\omega}{k} = \mp\frac{\lambda}{T} = \mp f\lambda$$

Traveling waves on a stretched string

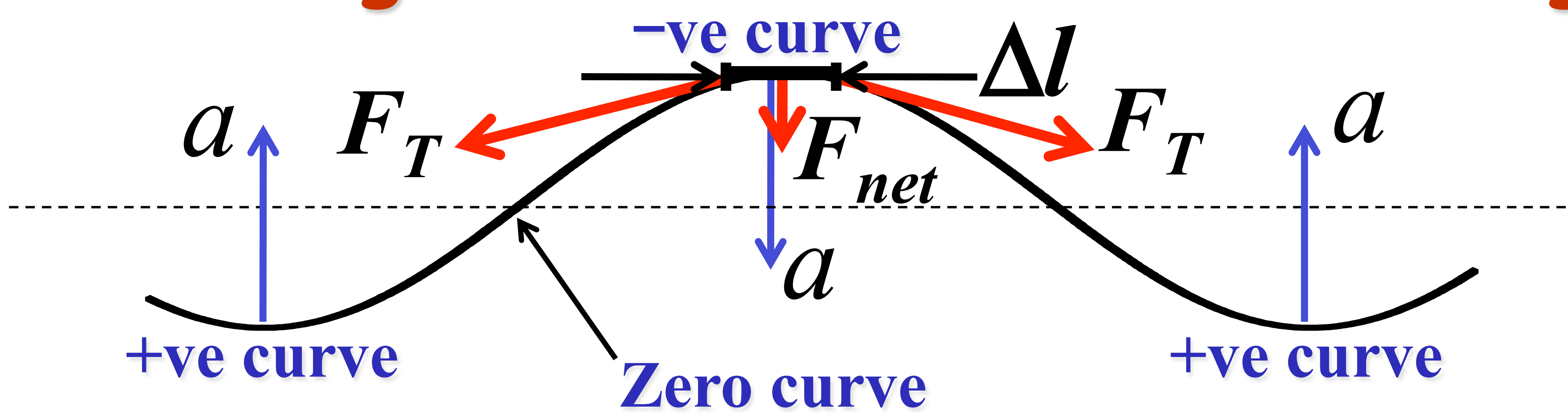


μ is the string's linear density, or mass per unit length.

- Tension F_T provides the restoring force ($kg.m.s^{-2}$) in the string. Without tension, the wave could not propagate.
- The mass per unit length μ ($kg.m^{-1}$) determines the response of the string to the restoring force (tension), through Newton's 2nd law.
- Look for combinations of F_T and μ that give dimensions of speed ($m.s^{-1}$).

$$v = \sqrt{\frac{F_T}{\mu}}$$

Traveling waves on a stretched string



μ is the string's linear density, or mass per unit length.

The Wave Equation

transverse acceleration

mass

$$F_{net} = F_T\left(\frac{\partial^2 y}{\partial x^2}\times \cancel{\Delta l}\right) = \left(\mu\times\cancel{\Delta l}\right)\frac{\partial^2 y}{\partial t^2} = ma_y$$

Dimensionless parameter proportional to curvature

$$\boxed{F_T\frac{\partial^2 y}{\partial x^2} = \mu\frac{\partial^2 y}{\partial t^2}}$$

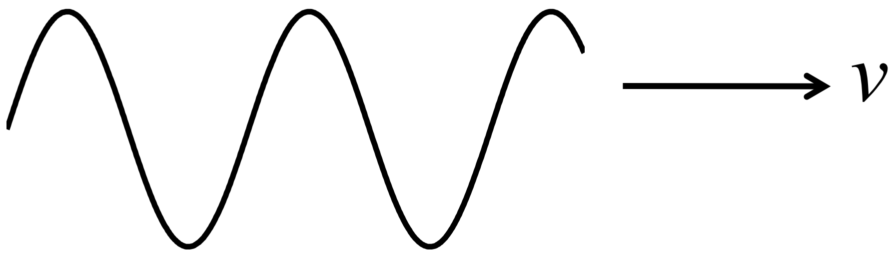
2nd Order Partial Differential Equation

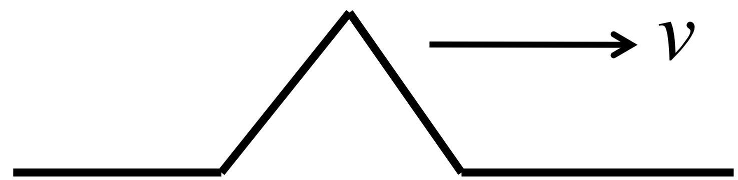
The wave equation

$$\frac{F_T}{\mu}\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

- General solution:

$$y(x,t) = \mathrm{A}\sin\left(kx \pm \omega t\right) \quad \text{or} \quad y(x,t) = \mathrm{A}\times f^{n}\left(kx \pm \omega t\right)$$





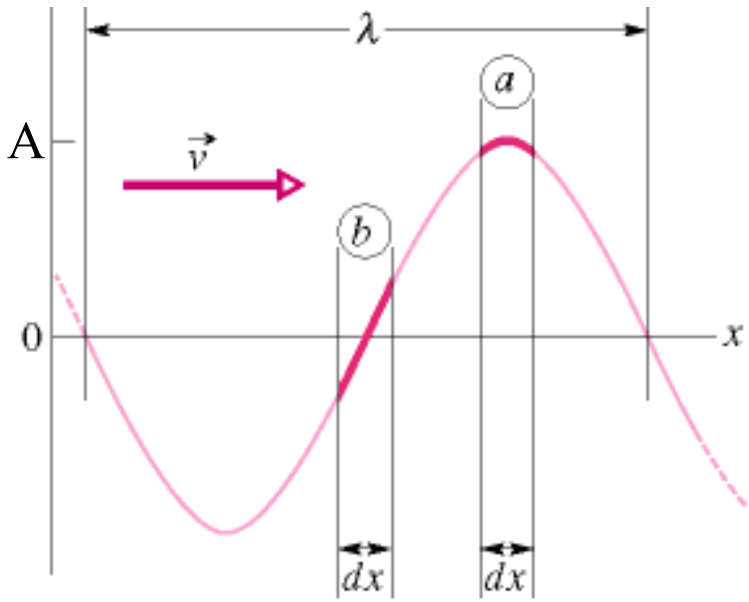
$$\frac{\partial^2 y}{\partial x^2} = -k^2 y\left(x,t\right) \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 y\left(x,t\right)$$

$$-\frac{F_T}{\mu}k^2 = -\omega^2 \quad \text{or} \quad \frac{\omega^2}{k^2} = v^2 = \frac{F_T}{\mu}$$

$$\boxed{\Rightarrow \quad v = \sqrt{\frac{F_T}{\mu}}}$$

Energy in traveling waves

$$y(x,t) = A\sin\left(kx - \omega t\right)$$



$$dm = \mu dx$$

Kinetic energy: $dK = {}^{1}/_{2}\, dm\, v_y^{\,2}$

$$v_y = \frac{\partial y}{\partial t} = -\omega A\cos\left(kx - \omega t\right)$$

$$dK = \tfrac{1}{2}\left(\mu dx\right)\left(-\omega A\right)^2 \cos^2(kx - \omega t)$$

Divide both sides by dt, where $dx/dt = v_x$

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v_x \omega^2 A^2 \cos^2(kx - \omega t)$$

Similar expression for elastic potential energy

$$\frac{dU}{dt} = \tfrac{1}{2}\mu v_x \omega^2 A^2 \cos^2(kx - \omega t)$$

$$\boxed{P_{avg} = 2\times\tfrac{1}{2}\mu v \omega^2 A^2 \left\langle \cos^2(kx - \omega t)\right\rangle = 2\times\tfrac{1}{2}\mu v\omega^2 A^2 \times \tfrac{1}{2} = \tfrac{1}{2}\mu v \omega^2 A^2}$$

Energy is pumped in an oscillatory fashion down the string

Note: I dropped the subscript on v since it represents the wave speed

The principle of superposition for waves

- It often happens that waves travel simultaneously through the same region, *e.g.*
  - Radio waves from many broadcasters
  - Sound waves from many musical instruments
  - Different colored light from many locations from your TV
- Nature is such that all of these waves can exist without altering each others' motion
- Their effects simply add
- This is a result of the principle of superposition, which applies to all harmonic waves, *i.e.*, waves that obey the linear wave equation

$$v^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

- And have solutions: $y(x,t) = \mathrm{A} \times f^n\left(kx \pm \omega t\right)$ or $\mathrm{A}\sin\left(kx \pm \omega t\right)$

The principle of superposition for waves

- If two waves travel simultaneously along the same stretched string, the resultant displacement y' of the string is simply given by the summation

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

where y_1 and y_2 would have been the displacements had the waves traveled alone.

- This is the principle of superposition.

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

Overlapping waves do not in any way alter the travel of each other

Interference of waves

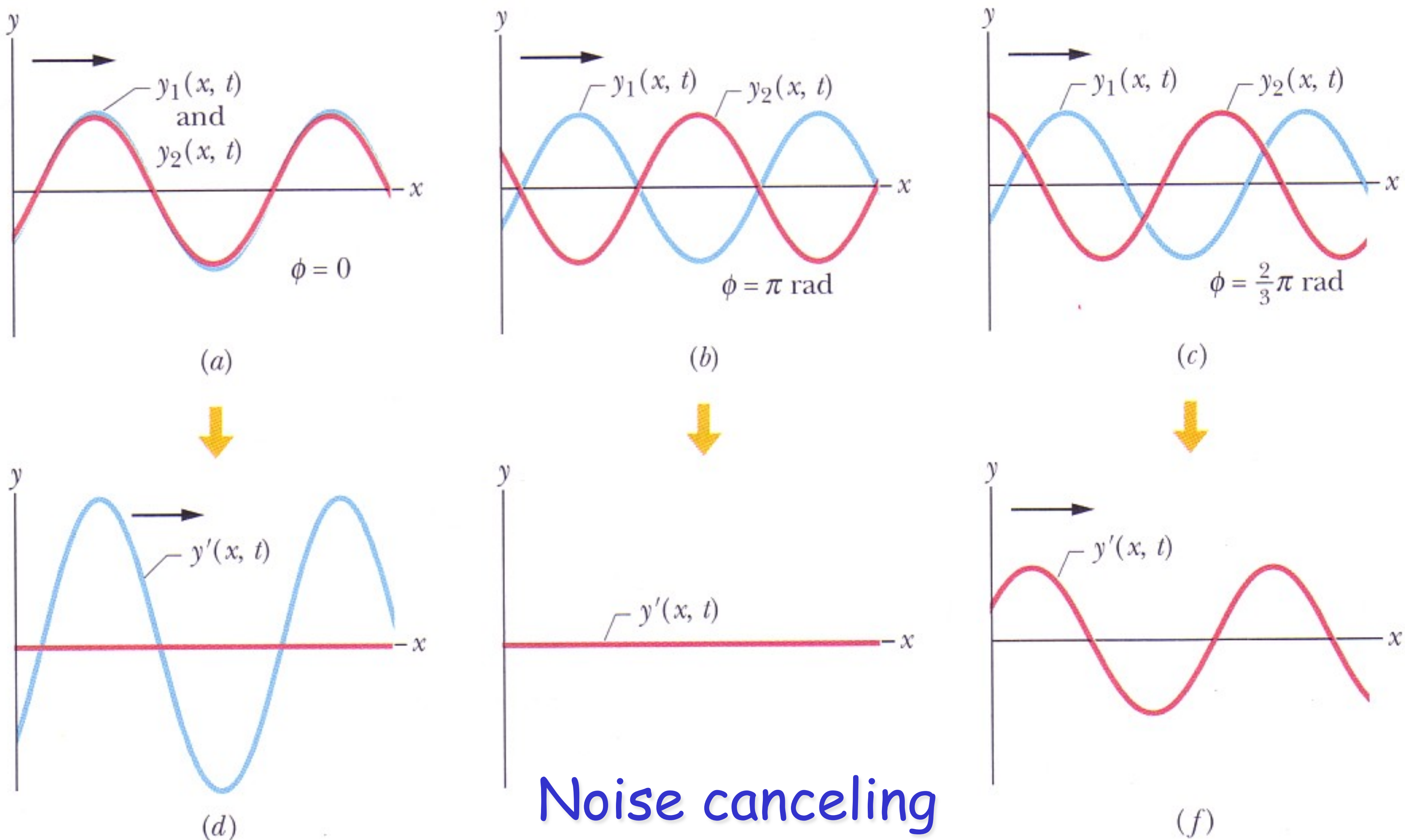
- Suppose two sinusoidal waves with the same frequency and amplitude travel in the same direction along a string, such that

$$y_1 = A\sin\left(kx - \omega t\right)$$

$$y_2 = A\sin\left(kx - \omega t + \phi\right)$$

- The waves will add.

Interference of waves



Noise canceling headphones

Interference of waves

- Suppose two sinusoidal waves with the same frequency and amplitude travel in the same direction along a string, such that

$$y_1 = \mathrm{A}\sin\left(kx - \omega t\right)$$

$$y_2 = \mathrm{A}\sin\left(kx - \omega t + \phi\right)$$

- The waves will add.
- If they are in phase (*i.e.* $\varphi = 0$), they combine to double the displacement of either wave acting alone.
- If they are out of phase (*i.e.* $\varphi = \pi$), they combine to cancel everywhere, since $\sin(\alpha) = -\sin(\alpha + \pi)$.
- This phenomenon is called interference.

Interference of waves

• Mathematical proof:

$$y_1 = \mathrm{A}\sin\left(kx - \omega t\right)$$

$$y_2 = \mathrm{A}\sin\left(kx - \omega t + \phi\right)$$

Then:

$$y'\left(x,t\right) = y_1\left(x,t\right) + y_2\left(x,t\right)$$

$$= \mathrm{A}\sin\left(kx - \omega t\right) + \mathrm{A}\sin\left(kx - \omega t + \phi\right)$$

But:

$$\sin\alpha + \sin\beta = 2\cos\tfrac{1}{2}\left(\alpha - \beta\right)\sin\tfrac{1}{2}\left(\alpha + \beta\right)$$

Phase shift

So:

$$y'\left(x,t\right) = \underbrace{\left[2\mathrm{A}\cos\tfrac{1}{2}\phi\right]}_{\textbf{Amplitude}}\underbrace{\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)}_{\textbf{Wave part}}$$

Interference of waves

$$y'(x,t) = \left[2A\cos\tfrac{1}{2}\phi\right]\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)$$

If two sinusoidal waves of the same amplitude and frequency travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in the same direction.

- If $\varphi = 0$, the waves interfere constructively, $\cos\frac{1}{2}\varphi = 1$ and the wave amplitude is $2A$.
- If $\varphi = \pi$, the waves interfere destructively, $\cos(\pi/2) = 0$ and the wave amplitude is 0, *i.e.*, no wave at all.
- All other cases are intermediate between an amplitude of 0 and $2A$.
- Note that the phase of the resultant wave also depends on the phase difference.

Adding waves as vectors (phasors) described by amplitude and phase

Wave interference - spatial